where

$$\mu = \frac{4\pi G}{c^2}\int_0^R \rho(r)\, r^2\, dr = \frac{4\pi G}{3c^2}(\rho R^3) \tag{14}$$

The element g_{44} of the metric (13) is given by the expression

$$\left(-1 + \frac{C}{r} + \frac{1}{3}\Lambda r^2\right)$$

where C is the same constant as in equation (3). The Schwarzschild radius of the universe is therefore determined by the conditions

$$g_{44}\left(r \gtrless R_S\right) \lessgtr 0 \tag{15}$$

which imply that

$$\left(-1 + \frac{C}{R_S} + \frac{1}{3}\Lambda R_S^2\right) = 0 \tag{16}$$

and

$$\left(\frac{\partial g_{44}}{\partial r}\right)_{r=R_S} = \left(-\frac{C}{R_S^2} + \frac{2}{3}\Lambda R_S\right) < 0 \tag{17}$$

Conditions (16) and (17) for R_S are identical with conditions (7) and (8) for R_{max}. This is true only for $\kappa = +1$, which therefore seems the natural choice. Solutions for R_S are therefore identically the same as for R_{max}—irrespective of the values of the parameters C and Λ, except that for $\Lambda > \Lambda_c$ neither R_S nor R_{max} exists. Now, the Schwarzschild radius of the universe is obtained from the static metric in the exterior of the universe while the maximal value of the function $R(t)$ was obtained from the nonstatic metric in the interior of the universe. The fact that the two, whenever they exist, are identically equal can hardly be a coincidence. It is therefore tempting to suggest that the identity $R_S = R_{max}$ is fundamental to the structure of the universe; accordingly, we must have

$$\kappa = +1 \text{ and } \Lambda \leq \Lambda_c \tag{18}$$

It follows that at any epoch

$$R(t) \leq R_S \tag{19}$$

which means that the universe is indeed in a black hole.

At present, the largest sphere that can be drawn in the universe has a surface area $4\pi R_0^2$. As expansion goes on, this may approach the maximal value $4\pi R_{max}^2$ ($\equiv 4\pi R_S^2$). Being inside a black hole, we cannot hope to "shoot through" the Schwarzschild surface; we may approach it in infinite time ($\Lambda = \Lambda_c$) or in a finite time ($\Lambda < \Lambda_c$). In the latter case, the universe must retrace its steps and proceed along a phase of contraction, eventually producing densities where present understanding of physics breaks down (with, perhaps, a further expansion from the "primaeval" matter and so on, in an endless cycle of pulsations).

Some of the immediate consequences of this picture may be expressed in terms of inequalities which must be satisfied by the various parameters characterizing the universe. Arising from conditions (18), these inequalities provide lower and upper bounds for the parameters Λ and ρ_0, and a lower bound for the parameter R_0, in terms of the observable quantities H_0 and q_0; so these parameters can be estimated from the kinematics of the universe alone. The numerical values resulting from these inequalities may not be very accurate because of the errors in the observed values of H_0 and q_0 (which, at the present time, may be as large as 20–40% (ref. 6)). But using $H_0 = 75$ km s^{-1} (Mpc^{-1}) and $q_0 = 1$ as "representative values"

$$-6.7 \times 10^{-57}\ \text{cm}^{-2} < \Lambda \leq \Lambda_c \leq 1.0 \times 10^{-57}\ \text{cm}^{-2} \tag{20}$$

$$1.5 \times 10^{-29}\ \text{g cm}^{-3} < \rho_0 \leq 2.3 \times 10^{-29}\ \text{g cm}^{-3} \tag{21}$$

$$R_0 \geq 1.1 \times 10^{28}\ \text{cm} \tag{22}$$

The ranges in which Λ and ρ_0 may lie are narrow.

Apart from these immediate consequences, there are deeper implications as well. For instance, we are now faced with several questions: How did the universe come to be a black hole—through a gravitational collapse, followed by a phase of expansion? In the cosmos, which includes the exterior as well as the interior of the universe, can our universe be unique? If not, what would its status be *vis-a-vis* other such structures in the cosmos? Investigation of these and other related questions, including the possible existence of an hierarchy of black holes, is clearly a matter of some importance.

As for our own universe, the concept of the Schwarzschild radius itself seems to be of considerable significance. Its relevance to other realms of physical phenomena will be discussed in a subsequent communication.

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1 Einstein, A., *Berlin Sitzungsberichte*, 142 (1917). English translation in *The Principle of Relativity* (Methuen, London, 1923).
2 *Nature* (*News and Views*), **232**, 440 (1971).
3 Robertson, H. P., and Walker, A. G., *Pub. Astron. Soc. Pacific*, **67**, 82 (1955).
4 Pathria, R. K., *The Theory of Relativity* (Hindustan Publishing Corporation, Delhi, 1963).
5 Robertson, H. P., and Noonan, T. W., *Relativity and Cosmology* (W. B. Saunders, Philadelphia, 1968).
6 Sandage, A. R., *Observatory*, **88**, 91 (1968).
7 Rindler, W., *Essential Relativity* (Van Nostrand Reinhold Co., New York, 1969).

Orbital Eccentricity of Mercury and the Origin of the Moon

A NUMBER of mechanisms for the formation of the Moon have been suggested; fission of the Earth, precipitation in a hot gaseous silicate atmosphere, independent formation in orbit about the Earth, and independent formation elsewhere in the solar system followed by capture by the Earth[1]. Although the last of these mechanisms has been admitted to be improbable by its proponents, they have shown that it is by no means impossible dynamically[2]. The principal objection to this mechanism is the strange composition of the Moon. It has been recognized for many years that the low mean density of the Moon implies that it is highly deficient in metallic iron. The lunar exploration programme has also shown that the Moon is much more deficient than the Earth in the more volatile of the condensable elements. Because of the apparent difficulty of satisfying these composition constraints in a theory in which the Moon is formed elsewhere in the solar system, I have tended to favour the other mechanisms mentioned above[3,4].

The situation now appears to be changed as a result of recent work by D. L. Anderson[5,6] (presented in most fully developed form at the IAGC). He justifies the postulate that, in addition to the above chemical abnormalities, the deep interior of the Moon is probably also very deficient in magnesium silicates, and that it is remarkably free of iron oxide in the silicate materials. Such a Moon would be close to the melting point in much of the deeper interior, yet its electrical conductivity would not exceed the bounds placed by analysis of lunar magnetometer measurements[7]. Anderson postulates that the Moon has the same basic composition as the calcium and aluminium-rich silicates in the inclusions in the Allende carbonaceous chondrite, which are believed to be very high temperature condensates within a cooling primitive solar nebula[8].

As a hot gas of solar composition cools, at a pressure of the order of 10^{-3} atmosphere, the first major constituents to condense consist of calcium, aluminium, and titanium oxides and silicates, such as corundum, perovskite, and melilite. As the gas continues to cool, metallic iron, alloyed with nickel, will be precipitated, and shortly thereafter magnesium silicates will condense[9].

I have recently constructed models of the primitive solar nebula, in which the internal temperatures and pressures are related by the adiabat likely to be produced on compression of the gas as it reaches the adiabatic state in the collapse of an interstellar gas cloud[10]. In such models, the temperature increases progressively toward the centre of the primitive solar nebula, and the interstellar grains are completely evaporated only near the central spin axis. I have also concluded that planetary bodies should accumulate very rapidly within such a primitive solar nebula[11]. In this picture it is a natural conclusion that the high mean density of the planet Mercury results from its accumulation from material in the primitive solar nebula in the temperature range in which metallic iron has condensed but the magnesium silicates have not. Lewis has independently reached this same conclusion[12].

Grossman[9] has pointed out that the calcium and aluminium-rich inclusions in the Allende meteorite can be formed inside the region of iron condensation in the primitive solar nebula, because their compositions are consistent with the very highest temperature condensates. If Anderson is correct in his conclusion that the bulk composition of the Moon resembles that of the Allende inclusions, then the natural place for the formation of the Moon in the solar system is inside the orbit of Mercury, through planetary accumulation from the condensed material to be found there.

This explains the anomalous large eccentricity (0.206) of the orbit of Mercury. The temperature gradient in the primitive solar nebula was probably steep enough for the orbital radii at which the Moon and Mercury were formed to differ by a much smaller relative amount than do the orbits of other neighbouring planets within the solar system. Thus gravitational perturbations of the orbits of the two bodies would probably accumulate until a close approach took place, at which a very large modification in the elements of the Moon's orbit would become possible. If the modified orbit of the Moon were sufficiently great to allow it to approach the Earth, then gravitational capture of the Moon by the Earth would become possible, even if improbable.

The semi-major axis of the orbit of Mercury is 0.387 astronomical units (a.u.). The eccentricity of this orbit is 0.206. Thus the aphelion distance of the orbit is at 0.467 a.u. It is reasonable to suppose that Mercury may have originally been in a circular orbit having a radius of 0.467 a.u.

The total energy, kinetic and potential, of a planet in orbit about the Sun is $GMm/2a$, where M is the mass of the Sun, m is the mass of the planet, and a is the semi-major axis of the orbit. In transforming from a circular orbit with $a = 0.467$ a.u. to the present orbit, the total energy of the orbit of Mercury would be decreased (algebraically) by 6.38×10^{38} erg. It is interesting to see what consequences would follow if this amount of energy should be transferred to the orbit of the Moon.

If the Moon started in a circular orbit at about 0.33 a.u. from the Sun, and was perturbed into an orbit having this distance as perihelion and an aphelion distance of about 1 a.u., then the semi-major axis of the transformed orbit will be about 0.67 a.u. The change of orbital energy required to produce this orbit is less than the above change in the orbital energy of Mercury. In fact, an initial circular orbit for the Moon could be situated as close to the Sun as 0.28 a.u. and be transformed into an orbit with a perihelion at this distance and an aphelion at the distance of the Earth.

Thus the chemical evidence pointing toward the formation of the Moon inside the orbit of Mercury is compatible with the dynamical requirement that the transformation of an initial circular orbit of Mercury into the present orbit of relatively large eccentricity should provide enough energy to transform the orbit of the Moon in such a way that the Moon crosses the orbit of the Earth.

The precise figures used in this analysis should be regarded as only illustrative. If the orbital perturbations occurred before the primitive solar nebula had been dissipated, the additional mass would modify the gravitational potential so that it was not quite inversely proportional to the distance from the central spin axis, and these figures would require modification. Also, even after dissipation of the primitive solar nebula, the Sun was probably much more massive than at present, because the T Tauri phase of mass loss would only be beginning. This would modify the numbers but not their relative orders of magnitude.

The above considerations pose a reasonably well defined problem for workers on the dynamical problem of capture of the Moon by the Earth, but they do not give its solution. The suggested orbit of the Moon, as perturbed by Mercury, is likely to be transformed further through perturbations by Venus and the Earth, but even so the approaches to the Earth will occur at a significant hyperbolic velocity. The most recent calculations on the dynamical capture process[2] have not yet determined the range of hyperbolic velocities which can be tolerated in this process.

The Earth probably acquired much of its present content of the more volatile elements by sweeping up smaller bodies in the vicinity of its orbit after the dissipation of the primitive solar nebula. Because the abundances of these elements in the Moon are so low, the capture of the Moon by the Earth probably did not occur until after this sweeping action of the Earth was almost complete.

As the lunar exploration programme has progressed, there has been an increasing tendency to regard the Moon as a planetary body in its own right. The conclusions above seem to justify this judgment.

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1 Kaula, W. M., *Revs. Geophys. Space Phys.*, **9**, 217 (1971).
2 Singer, S. F., *EOS. Trans. Am. Geophys. Union*, **51**, 637 (1970).
3 Cameron, A. G. W., in *The Earth–Moon System* (Marsden, B. G., and Cameron, A. G. W., eds.) (Plenum Press, New York, 1966).
4 Cameron, A. G. W., *EOS. Trans. Am. Geophys. Union*, **51**, 628 (1970).
5 Hanks, T. C., and Anderson, D. L., *Phys. Earth Planet Int.* (in the press).
6 Anderson, D. L., preprint, *IAGC Symp. Cosmochem.* (Cambridge, Mass., August 1972).
7 Sonett, C. P., Schubert, G., Smith, B. F., Schwartz, K., and Colburn, D. S., *Geochim. Cosmochim. Acta Suppl.*, **2**, 2415 (1971).
8 Marvin, U. B., Wood, J. A., and Dickey, jun., J. S., *Earth Plan. Sci. Lett.*, **7**, 346 (1970).
9 Grossman, L., *Geochim. Cosmochim. Acta*, **36**, 597 (1972).
10 Cameron, A. G. W., and Pine, M. R., *Numerical Models of the Primitive Solar Nebula, Icarus* (in the press).
11 Cameron, A. G. W., *Accumulation Processes in the Primitive Solar Nebula, Icarus* (in the press).
12 Lewis, J. S., *Earth Plan. Sci. Lett.*, **15**, 286 (1972).

Stratospheric Nitrogen Dioxide from Infrared Absorption Spectra

STRATOSPHERIC absorption due to NO_2 has been identified[1-3] in infrared spectra of the solar radiation recorded from balloon borne gondolas floating at mid-latitudes in the altitude range